Midterm 2 − Review − Problems

Peyam Ryan Tabrizian

Tuesday, November 12th, 2013

1 Derivatives

Problem 1

Find the slope of the tangent line to following the curve at $(2, 1)$:

$$\tan^{-1}\left(\frac{2}{x}\right) = \sin^{-1}\left(\sqrt{\frac{y}{2}}\right)$$

Problem 2

If $F(x) = f(3f(4f(x)))$, where $f(0) = 0, f'(0) = 2$, find $F'(0)$

2 Antiderivatives

Problem 3

A particle moves with an acceleration $a(t) = 6t$ ft/s^2. Its velocity at time $t = 0$ is 2 ft/s. What is the net change of position of the particle between times $t = 1$ and $t = 2$?

3 Exponential growth and decay

Problem 4

The half-life of cesium−137 is 30 years. Suppose we have a 100−mg sample. After how many years will only 1 mg remain?

4 Linear approximation

Problem 5

Use linear approximations (or differentials) to approximate $(2.013)^3$.

5 L'Hopital's rule

Problem 6

Evaluate the following limits

(a) $\lim_{x\to 0^+} \sin(x)\ln(x)$

(b) $\lim_{x\to 0^+} (\sin(x))^x$

6 Mean Value Theorem

Problem 7

Show that $x^5 - 6x = c$ has at **most** one solution in $[-1, 1]$

Problem 8

Is there a function f with $f(0) = -1$, $f(2) = 4$ and $f'(x) \le 2$ for all x?

7 Related rates

Problem 9

A cylindrical gob of goo is undergoing a transformation in which its height is decreasing by 1 cm per second, while its volume is decreasing by 2π cm^3 per second. If its volume at a certain instant is 24π cm^3 and its height is 6 cm, determine if its radius is increasing or decreasing at that instant, and at what rate.

8 Optimization

Problem 10

A woman is on the upper-left corner A of a rectangular lake that is 3 km wide and 8 km long, and her goal is to reach a point B on the lower-right corner in the shortest possible time. Suppose she can row southeast at a speed of 6 km/h to reach a point D on the right-hand-side of the lake, and then run down directly to B at a speed of 8 km/h. How should she proceed?

9 Graphing

Problem 11

Graph $y = \frac{\sin(x)}{1+\cos(x)}$